Turbulence:
The manifestation of eddies and their role in conservation laws

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What I Will Cover

- Motivation
- Fluid Motion
- The Boussinesq Approximation
- Fluid Modeling
- Boundary Layers
- Vorticity
- What Does Turbulence Do?
- How Do We Study Turbulence?
- Trends in Research
- Some Final Thoughts
- A Practical Example of Turbulence

Motivation

Why study fluids?

What is Turbulence?

When the flow of a fluid goes from being smooth to being broken, with weird curves, bifurcations, and other similar phenomena, we call it turblence. This is about as good a definition as you will get. As with many things, we may not be able to say exactly what it is, but we know it when we see it.

Why is it Interesting?

Fluid flow is either smooth (laminar) or unstable (turbulent). Any time you have a non-ideal fluid (which is always in the real world) the flow becomes turbulent at some point.

Fluid Motion

The governing equations for fluid flow are the momentum equation (effectively Newton’s second law), the continuity equation, the energy equation (the first law of thermodynamics),, and the equation of state.

Time – Rate of Change of Momentum =
 Density × Volume Force + Stress Rate of Strain (1)

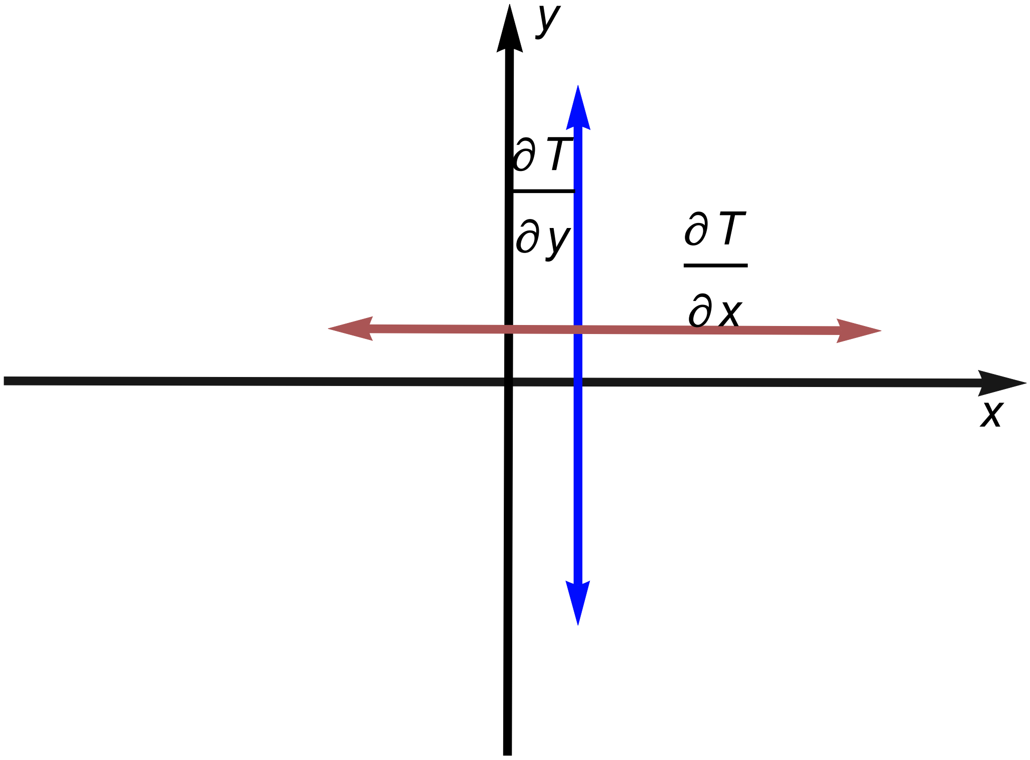
Time – Rate of Change of Density = Density × Divergence of the Velocity Field (2)

Temperature × Time – Rate of Change of Entropy =
 Divergence of the Velocity Field × (Stress Rate of Strain + Pressure) +
  Divergence of the product of the heat conduction
   coefficient and the gradient of the temperature field (3)

Pressure = Some Specified Function of the Density and Temperature (4)

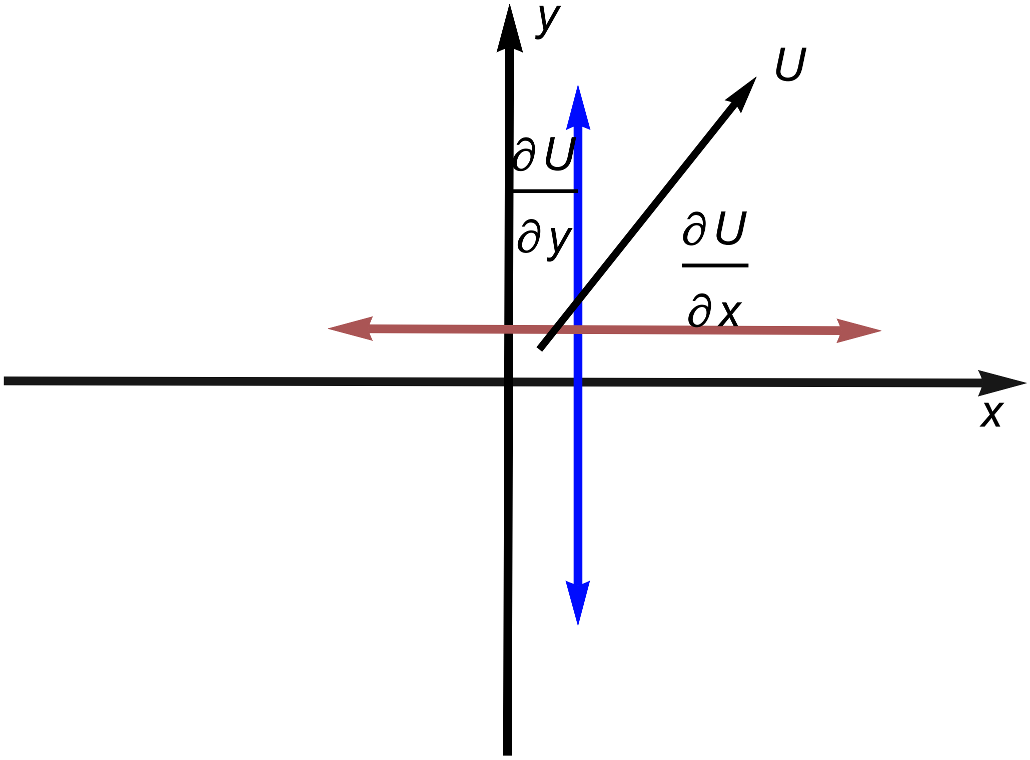
We understand a field to exist when every point of a region we are considering has a value for the quantity of concern (temeprature, velocity, stress, etc.).

The Temperature Field Gradient



The gradient of the temperature field is then a vector $\left(\frac{\partial T}{\partial x}, \frac{\partial T}{\partial y}\right)$

The Velocity Field Divergence



The divergence of the velocity vector field is the scalar $\frac{\partial U}{\partial x} + \frac{\partial U}{\partial y}$

The Boussinesq Approximation

One often assumes that the fluid is incompressible in order to make the equations tractible. This is only of limited value as every fluid is compressible to some level, (okay, maybe not the membrane interpretation of the event horizon of a black hole—but that is hardly a limiting case...)

This assumption allows us to consider density to be constant.

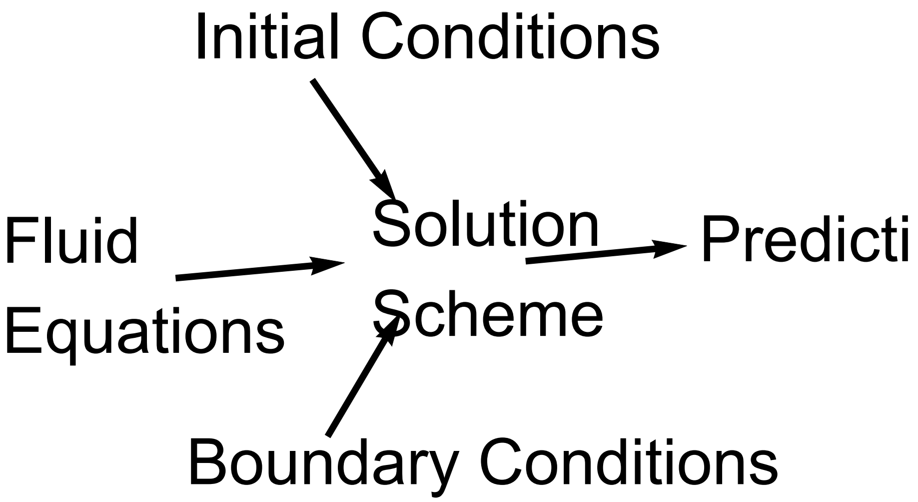
This is often called the Boussinesq approximation.

Incompressibility implies that in any volume of space, the quantity of fluid going into the volume is exactly the same as what flows out of it.

Surprisingly this is still adopted in many atmospheric models despite the ease of compression of air. While such an approximation does not invalidate a model, it must be justified on a case-by-case basis.

Fluid Modeling

A model of a fluid is a structure designed to predict the behavior of a fluid under definite circumstances.



Fluid Modeling

The combination of the equations and the initial and boundary conditions is a system.

The goal of a fluid model is to follow the time evolution of a system.

Fluid Modeling—Mathematical Modeling

The goal of mathematical modeling is to produce a mathematical scheme for solving a specific system.

A good portion of the volume of most books on Mathematical Methods for Science/Engineering is taken up with such schemes.

The advantage of such schemes is that they are both exact and continuous.

The disadvantage is that they are almost impossible to find—particularly for fluids.

Fluid Modeling—Computational Modeling

The goal here is to use a computer to perform a huge number of approximations to solve your system.

The advantage of this method is that just about any system can be solved in this way.

The disadvantage is that as time goes forward, the model will fill up with error—at some point we will lose all ability to predict accurately what will happen next.

Fluid Modeling—Two Approaches

The first approach is to imagine that a fluid is composed of blobs, small with respect to the fluid. We follow the motion of these blobs as time goes on, which we will call parcels. Sometimes they are called control volumes. This approach is useful for finding turbulence, but it is computationally intensive. This is called a Lagrangian approach.

The second approach is where we establish a grid system. At each intersection of the grid we solve our equations for each time step. This is computationally efficient. The problem is that things can (and will) happen in between the grid point that we will not be able to follow.

Fluid Modeling—The Lagrangian Approach

Reynolds Number

The first thing we need to characterize a flow around an object is the length of the object. When this is compared to the flow velocity we get a time scale in which a flow phenomenon will be carried across the object. We call this process advection. Any quantity can be advected, it means that that quantity is being carried by the fluid from one place to another.

$$\text{Time Scale for Advection} = \frac{\text{Length}}{\text{Velocity}} \tag{5}$$

We also need to determine the time it takes for a quantity to be mixed from one side of the object to the other. This mixing occurs through the interactions of the atoms and molecules interacting in the fluid. We have a name for this mixing, it is called viscous diffusion. We need to measure, or otherwise estimate, the viscosity—the transport of momentum by the collective motion of fluid particles.

$$\text{Time Scale for Viscous Diffusion} = \frac{\text{Length}^2}{\text{Viscosity}} \tag{6}$$

The ratio of these time scales is what we call the Reynolds Number,

$$\text{Reynolds Number} = \frac{\text{Time Scale for Viscous Diffusion}}{\text{Time Scale for Advection}} \tag{7}$$

Thus, the higher the Reynolds Number, the more viscosity (and hence friction).

Boundary Layers

As the Reynolds number increases, a thin layer forms over the surface of the object within the velocity field. In this layer viscosity is dominant. Thus momentum is dispersed throughout this layer. We can say that momentum is mixed in this layer. Outside of the layer, the flow behaves as if it were laminar. Inside we see a very different flow, one that becomes dominated by visosity to pull away from the laminar flow. In this flow, streamlines diverge from the laminar regime, and at some point they converge.

This has the effect of reducing the drag coefficient of an object in the fluid.

In the atmosphere, this process is complicated by the bouyancy of a parcel, and by the presence of a warm/dry layer of air aloft. If a parcel is warmer than its surroundings it experiences positive bouyancy and tends to rise. If a parcel is cooler, it experiences negative bouyancy and sinks. Parcels at the same temperature as its surroundings are stable, and experience no bouyancy.

A warm/dry layer of air aloft may stop parcels from rising through it. We call this a capping inversion. This may be thought of as the top of the atmospheric boundary layer.

Vorticity

If we think of the velocity as having an *x*-component, called *u*, and a *y*-component, called *v*, then we can write the velocity as (*u*, *v*). If there is a factor of *y* in *u*, and/or a factor of *x* in *v*, then we can define a new quantity,

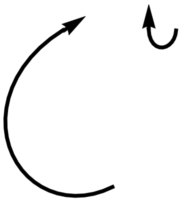
$$\text{vorticity} = x - \text{rate of change of } v - y - \text{rate of change in } u \tag{8}$$

If you know vector analysis, you will note that this is the z-component of the curl of the velocity field. The vorticity is the rate of curving in the fluid flow. As the Reynolds number increases we see that this quanity increases to where the flow actually curves around all the way. This circular flow is called a vortex. When the fluid flow reverses against the general motion of the flow we call that an eddy.

What Does Turbulence Do?

So, what does an irregular flow do?

- It drives energies from larger scale flows to smaller scales.



What Does Turbulence Do?

- When studying the velocity field, we will find oscillations. When these are sinusoidal we call them modes. By using Fourier analysis we split up the complicated velocity field motions into modes. It turns out that there will be coupling between different modes. So changes in one scale of motion will causes changes at other scales. This is called the weak-turbulence formalism.
- In strong turbulence we see that eddies of some size last no longer than the advection time-scale for that size. Thus we have eddies come into and go out of existence in a chaotic way. Thus energy gets redistributed in ways that are not predictable. This turbulent flow gets advected downstream.

What Does Turbulence Do?

- The Coanda effect causes wakes and jets to curve towards surfaces.
- Such wakes and jets capture fluid downstream, this is called entrainment.
- Wakes, because they have vorticity, become irregular downstream.
- Wakes dissipate energy in the form of heat, the rate of which is greates at the core of the wake.

What Does Turbulence Do?

- Causes the formation of turbulent boundary layers, and a consequent drag reduction.
- Changes the rate of chemical reactions.
- Properties are mixed efficiently.
- Backscatter, energy being driven from small eddies to larger scales.

How Do We Study Turbulence?

- Experimental/Observational Studies
- DNS
- Statistical Modeling
- Fourier Analysis
- Perturbation Methods
- Reynolds-Averaged Navier Stokes (RANS)
- Large-Eddy Simulation (LES)
- Topological Approach
- Fractal Approach
- Sum Over Paths
- Active Research

Trends in Research

We have seen an evolution of ideas.

Our understanding began as observational leading to a statistical treatment.

This evolved into a structural treatment.

This has further evolved into a deterministic treatment.

Where Do Things Stand?

We know what turbulence does.

We think we know why.

We will never be able to predict it.

We do not, now, know how to cope with it.

References

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A Practical Example

Thank You!