Lecture 6: Exercise 1

Explanation

This exercise is designed to demonstrate how equations that appear vastly different can, in fact, be the same.

Hint

Examine each component of the equations and determine what they mean.

Answer

We begin be writing Equation (4),

$$\frac{d}{d\,t}\left(\frac{\partial L}{\partial \dot{x}}\right) - \frac{\partial L}{\partial x} = 0$$

We can rewrite this

$$\frac{d}{d\,t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x}$$

Since

$$L = T + V = \frac{1}{2} m \dot{x}^2 + V(x)$$

then

$$\frac{\partial L}{\partial x} = \frac{\partial}{\partial x} V(x)$$

from the definition of force, this is F from Newton's equation of motion. Then

$$\frac{\partial L}{\partial \dot{x}} = 2 \times \frac{1}{2} m \dot{x} = m \dot{x}$$

this is the canonical momentum. Taking its time derivative,

$$\frac{d}{d t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{d}{d t} m \dot{x} = m \ddot{x} = m a$$

so, we have

$$\frac{d}{d t}\left(\frac{\partial L}{\partial \dot{x}}\right) = \frac{\partial L}{\partial x} \Longrightarrow m a = F$$

thus Lagrange's equations and Newton's equation are the same.