

# Lecture 5: Exercise 1

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## Explanation

This exercise establishes a general rule for derivatives: When differentiating a function  $f(t)$  with respect to  $t$ , always end the expression with  $\frac{d f(t)}{d t} = f'(v) \frac{d v}{d t}$  whenever  $f$  can be written as a function of  $v$ .

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## Hint

Recall the product rule and the chain rule.

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## Answer

We begin by writing,

$$\frac{d}{d t} v^2 = \frac{d}{d t} (v v)$$

we apply the product rule

$$\frac{d}{dt} v^2 = v \frac{dv}{dt} + v \frac{dv}{dt} = 2v \frac{dv}{dt} = 2v \dot{v}.$$

Note that this is the same as writing

$$\frac{d}{dt} v^n = n v^{n-1} \frac{dv}{dt}.$$

This can be viewed as a kind of application of the chain rule, where  $f(t) = f(v)$

$$\frac{df(t)}{dt} = f'(v) \frac{dv}{dt}.$$