Lecture 5: Exercise 1

**Explanation**

This exercise establishes a general rule for derivatives: When differentiating a function $f(t)$ with respect to $t$, always end the expression with

$$\frac{df(t)}{dt} = f'(v) \frac{dv}{dt}$$

whenever $f$ can be written as a function of $v$.

**Hint**

Recall the product rule and the chain rule.

**Answer**

We begin by writing,

$$\frac{d}{dt} v^2 = \frac{d}{dt} (v \cdot v)$$

we apply the product rule
\[
\frac{d}{dt} v^2 = v \frac{dv}{dt} + v \frac{dv}{dt} = 2 v \frac{dv}{dt} = 2 v v.
\]

Note that this is the same as writing
\[
\frac{d}{dt} v^a = v^{a-1} \frac{dv}{dt}.
\]

This can viewed as a kind of application of the chain rule, where \( f(t) = f(v) \)
\[
\frac{d f(t)}{dt} = f'(v) \frac{dv}{dt}.
\]