Lecture 3: Exercise 4

Explanation

This exercise provides some more practice in proving that a proposed solution to a differential equation is indeed a solution. This also establishes the general solution of the harmonic oscillator problem.

Hint

You will need the sum rule and the trigonoemtric rules of differentiation.

Answer

We begin by writing,

$$x(t) = A \cos \omega t + B \sin \omega t.$$

We can find the velocity by differentiating,

$$v(t) = \frac{d}{d t} x(t) = \frac{d}{d t} A \cos \omega t + \frac{d}{d t} B \sin \omega t$$

or,

$$v(t) = A \frac{d}{d\,t} \cos \omega\, t + B \frac{d}{d\,t} \sin \omega\, t$$

or,

$$v(t) = -A\, \omega \sin \omega\, t + B\, \omega \cos \omega\, t.$$

We can find the acceleration,

$$a(t) = \frac{d}{d\,t} v(t) = -\frac{d}{d\,t} A\, \omega \sin \omega\, t + \frac{d}{d\,t} B\, \omega \cos \omega\, t$$

or,

$$a(t) = -A\, \omega \frac{d}{d\,t} \sin \omega\, t + B\, \omega \frac{d}{d\,t} \cos \omega\, t$$

or

$$a(t) = -A\, \omega^2 \cos \omega\, t - B\, \omega^2 \sin \omega\, t$$

we can factor the ω^2,

$$a(t) = -(A \cos \omega\, t - B \sin \omega\, t)\, \omega^2 = -\omega^2\, x(t),$$

which is the form of Eq. (6). When $t = 0$,

$$x(0) = A \cos 0 + B \sin 0 = A.$$

and

$$v(0) = -A\, \omega \sin 0 + B\, \omega \cos 0 = B\, \omega.$$

If we label the initial position v_0 and the initial velocity v_0, then

$$x_0 = A$$

and

$$v_0 = B\, \omega.$$