Lecture 3: Exercise 3

Explanation

This exercise provides some practice in proving that a proposed solution to a differential equation is indeed a solution.

Hint

Recall the sum and power rules of differentiation. You will need to differentiate twice.

Answer

We begin by writing,

$$z(t) = z_0 + v_z(0)\, t + \frac{F_z}{2\, m}\, t^2.$$

We can find the velocity by differentiating,

$$v_z(t) = \frac{d}{d\, t}\, z(t) = \frac{d}{d\, t}\, z_0 + \frac{d}{d\, t}\, v_z(0)\, t + \frac{d}{d\, t}\, \frac{F_z}{2\, m}\, t^2.$$

or,

$$v_z(t) = v_z(0)\,\frac{d}{d\,t}\,t + \frac{F_z}{2\,m}\,\frac{d}{d\,t}\,t^2.$$

or,

$$v_z(t) = v_z(0)\times 1 + \frac{F_z}{2\,m}\,2\,t = v_z(0) + \frac{F_z}{m}\,t$$

We can find the acceleration,

$$a_z(t) = \frac{d}{d\,t}\,v_z(t) = \frac{d}{d\,t}\,v_z(0) + \frac{d}{d\,t}\,\frac{F_z}{m}\,t$$

or,

$$a_z(t) = \frac{d}{d\,t}\,v_z(0) + \frac{F_z}{m}\,\frac{d}{d\,t}\,t = \frac{F_z}{m}$$

or

$$m\,a_z(t) = F_z.$$

Which is Newton's equation of motion.