

Lecture 3: Exercise 2

Explanation

This exercise provides more practice for integrating a differential equation.

Hint

Recall the definition of definite integrals and acceleration.

Answer

We begin by writing,

$$\dot{v}_z = \frac{F_z}{m}$$

or

$$\frac{d v_z}{d t} = \frac{F_z}{m}$$

we separate the variables

$$d v_z = \frac{F_z}{m} d t$$

and we integrate from v_{z_0} to $v_z(t)$, and from $t'=0$ to t

$$\int_{v_{z_0}}^{v_z(t)} d v_z' = \int_0^t \frac{F_z}{m} d t'$$

or

$$\int_{v_{z_0}}^{v_z(t)} d v_z' = \frac{F_z}{m} \int_0^t d t'$$

evaluating this we have

$$v_z \Big|_{v_{z_0}}^{v_z(t)} = \frac{F_z}{m} t' \Big|_0^t$$

or

$$v_z(t) - v_{z_0} = \frac{F_z}{m} t$$

or

$$v_z(t) = \frac{F_z}{m} t + v_{z_0}.$$