Lecture 3: Exercise 2

Explanation
This exercise provides more practice for integrating a differential equation.

Hint
Recall the definition of definite integrals and acceleration.

Answer
We begin by writing,

\[ v_z = \frac{F_z}{m} \]

or

\[ \frac{d v_z}{d t} = \frac{F_z}{m} \]
we separate the variables

\[ d v_z = \frac{F_z}{m} d t \]

and we integrate from \( v_{z0} \) to \( v_z(t) \), and from \( t'=0 \) to \( t \)

\[ \int_{v_{z0}}^{v_z(t)} d v_z' = \int_{0}^{t} \frac{F_z}{m} d t' \]

or

\[ \int_{v_{z0}}^{v_z(t)} d v_z' = \frac{F_z}{m} \int_{0}^{t} d t' \]

evaluating this we have

\[ v_z \bigg|_{v_{z0}}^{v_z(t)} = \frac{F_z}{m} \bigg|_{0}^{t} \]

or

\[ v_z(t) - v_{z0} = \frac{F_z}{m} t \]

or

\[ v_z(t) = \frac{F_z}{m} t + v_{z0}. \]