

Lecture 2: Exercise 7

Explanation

In this exercise we are carrying the discussion of circular motion to a new mathematical level. We are proving that the position vector and velocity vector are perpendicular, orthogonal.

Hint

Recall the definition of orthogonality, that a dot product of orthogonal vectors is 0.

Answer

The dot product of the vectors has to be zero, so,

$$\begin{aligned}\vec{r} \cdot \vec{v} &= (R \cos \omega t) (-R \omega \sin \omega t) + (R \sin \omega t) (R \omega \cos \omega t) \\ &= R^2 \omega \sin \omega t \cos \omega t - R^2 \omega \sin \omega t \cos \omega t \\ &= R^2 \omega \sin \omega t \cos \omega t (1 - 1) \\ &= 0.\end{aligned}$$

The position and velocity vectors must be orthogonal.