Lecture 2: Exercise 3

Explanation

This is an application of the chain rule, which really simplifies taking derivatives of complicated expressions. The idea is to convert complicated sub-expressions into symbols and then do the differentiation.

Hint

Choose a single variable for a sub-expression and then apply the chain rule.

Answer

- $\boldsymbol{g(t) = \sin\left(t^2\right) - \cos\left(t^2\right)}$**.**

The first choice is the sub-expression to convert to a variable. We will choose to write $u = t^2$, then we have

$$g(u) = \sin u - \cos u.$$

The chain rule in this case is written

$$g'(t) = g'(u)\, u'(t).$$

So,

$$u'(t) = 2t$$

and,

$$g'(u) = \cos u + \sin u$$

then,

$$g'(t) = 2t(\cos u + \sin u) = 2t\cos u + 2t\sin u.$$

Since $u = t^2$, then

$$g'(t) = 2t\cos t^2 + 2t\sin t^2.$$

- **$\theta(\alpha) = e^{3\alpha} + 3\alpha \ln(3\alpha)$.**

We begin by deciding to write $u = 3\alpha$. So we have

$$\theta(u) = e^u + u\ln u.$$

Then we have

$$\theta'(u) = e^u + 1 + \ln u$$

and

$$u'(\alpha) = 3$$

so,

$$\theta'(\alpha) = \theta'(u)\,u'(\alpha) = 3e^u + 3 + 3\ln u = 3e^{3\alpha} + 3 + 3\ln(3\alpha).$$

- **$x(t) = \sin^2(t^2) - \cos(t^2)$.**

Again we write $u = t^2$, so

$$x(u) = \sin^2 u - \cos u.$$

We also have

$$x'(u) = 2\sin u\cos u + \sin u$$

and

$$u'(t) = 2t$$

thus

$$x'(t) = x'(u)\,u'(t) = 2t(2\sin u\cos u + \sin u) = 4t\sin t^2\cos t^2 + 2t\sin t^2.$$