Lecture 2: Exercise 2

Explanation
This is a straightforward extension of the idea of a derivative, applied to a derivative.

Hint
Apply the definition of differentiation step-by-step to the results of the previous exercise.

Answer
- \( f(t) = t^4 + 3t^3 - 12t^2 + t - 6. \)
  The first derivative was,
  \[ f'(t) = 4t^3 + 9t^2 - 24t + 1. \]
  We use the power rule to get
  \[ f''(t) = 12t^2 + 18t - 24. \]
\[ g(x) = \sin x - \cos x. \]
The first derivative is,
\[ g'(x) = \cos x + \sin x. \]
Here we again use the sum rule and the results of Eq. (2), this leads to an interesting result: the second derivative is the initial function with a sign change:
\[ g''(x) = -\sin x + \cos x = -g(x). \]

\[ \theta(\alpha) = e^\alpha + \alpha \ln \alpha. \]
The first derivative is,
\[ \theta'(\alpha) = e^\alpha + 1 + \ln \alpha. \]
We again use the sum rule
\[ \theta''(\alpha) = \frac{d}{d\alpha} e^\alpha + \frac{d}{d\alpha} 1 + \frac{d}{d\alpha} \ln \alpha. \]
We then apply Eq. (2) for the first term,
\[ \theta''(\alpha) = e^\alpha + \frac{d}{d\alpha} 1 + \frac{d}{d\alpha} \ln \alpha. \]
The derivative of a constant is always zero, since a constant does not change,
\[ \theta''(\alpha) = e^\alpha + \frac{d}{d\alpha} \ln \alpha. \]
We then apply Eq. 2 for the final term,
\[ \theta''(\alpha) = e^\alpha + \frac{1}{\alpha}. \]

\[ x(t) = \sin^2 t - \cos t. \]
The first derivative is,
\[ x'(t) = 2 \sin t \cos t + \sin t. \]
Here we again apply the sum rule,
\[ x''(t) = \frac{d}{dt} 2 \sin t \cos t + \frac{d}{dt} \sin t. \]
We again complete the second term, as it is simpler,
\[ x''(t) = -2 \sin t \cos t + \cos t. \]
We are left with the first term, which we expand using the product rule,
\[ \frac{d}{dt} 2 \sin t \cos t = \sin t \cos t \frac{d}{dt} 2 + \cos t \frac{d}{dt} \sin t + 2 \sin t \frac{d}{dt} \cos t = 0 + 2 \cos^2 t - 2 \sin^2 t = 2(\sin^2 t - \cos^2 t) = 2 \cos(2t) \]
So this gives us
\[ x''(t) = 2 \cos 2t + \cos t. \]