Lecture 2: Exercise 2

Explanation

This is a straightforward extension of the idea of a derivative, applied to a derivative.

Hint

Apply the definition of differentiation step-by-step to the results of the previous exercise.

Answer

- **$f(t) = t^4 + 3\,t^3 - 12\,t^2 + t\, - 6.$**

The first derivative was,

$$f'(t) = 4\,t^3 + 9\,t^2 - 24\,t + 1.$$

We use the power rule to get

$$f''(t) = 12\,t^2 + 18\,t - 24.$$

- **$g(x) = \sin x - \cos x$.**

The first derivative is,

$$g'(x) = \cos x + \sin x.$$

Here we again use the sum rule and the results of Eq. (2), this leads to an interesting result: the second derivative is the initial function with a sign change:

$$g'(x) = -\sin x + \cos x = -g(x).$$

- **$\theta(\alpha) = e^{\alpha} + \alpha \ln \alpha$.**

The first derivative is,

$$\theta'(\alpha) = e^{\alpha} + 1 + \ln \alpha.$$

We again use the sum rule

$$\theta''(\alpha) = \frac{d}{d\alpha} e^{\alpha} + \frac{d}{d\alpha} 1 + \frac{d}{d\alpha} \ln \alpha.$$

We then apply Eq. (2) for the first term,

$$\theta''(\alpha) = e^{\alpha} + \frac{d}{d\alpha} 1 + \frac{d}{d\alpha} \ln \alpha.$$

The derivative of a constant is always zero, since a constant does not change,,

$$\theta''(\alpha) = e^{\alpha} + \frac{d}{d\alpha} \ln \alpha.$$

We then apply Eq. 2 for the final term,

$$\theta''(\alpha) = e^{\alpha} + \frac{1}{\alpha}.$$

- **$x(t) = \sin^2 t - \cos t$.**

The first derivative is,

$$x'(t) = 2 \sin t \cos t + \sin t.$$

Here we again apply the sum rule,

$$x''(t) = \frac{d}{dt} 2 \sin t \cos t + \frac{d}{dt} \sin t.$$

We again complete the second term, as it is simpler,

$$x''(t) = \frac{d}{dt} 2 \sin t \cos t + \cos t.$$

We are left with the first term, which we expand using the product rule,

$$\begin{aligned} \frac{d}{dt} 2 \sin t \cos t &= \sin t \cos t \frac{d}{dt} 2 + 2 \cos t \frac{d}{dt} \sin t + 2 \sin t \frac{d}{dt} \cos t \\ &= 0 + 2\cos^2 t - 2 \sin^2 t = 2\left(\sin^2 t - \cos^2 t\right) = 2 \cos(2t) \end{aligned}$$

So this gives us

$$x''(t) = 2 \cos 2t + \cos t.$$