

Interlude 2: Exercise 4

Explanation

This exercise completes the derivation of the result of using integration by parts.

Hint

Recall the integration formulas for trigonometric functions.

Answer

We have

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx = \frac{\pi}{2} \sin \frac{\pi}{2} - \int_0^{\frac{\pi}{2}} \sin x \, dx$$

so,

$$\int_0^{\frac{\pi}{2}} \sin x \, dx = \cos x \Big|_0^{\frac{\pi}{2}} = \cos \frac{\pi}{2} - \cos 0 = \cos \frac{\pi}{2} - 1$$

thus,

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx = \frac{\pi}{2} \sin \frac{\pi}{2} - \cos \frac{\pi}{2} + 1.$$

Now, $\sin \frac{\pi}{2} = 1$, and $\cos \frac{\pi}{2} = 0$, so

$$\int_0^{\frac{\pi}{2}} x \cos x \, dx = \frac{\pi}{2} + 1.$$