Interlude 2: Exercise 1

Explanation

This is an exercise in integration. The idea is to be able to look at an integeral and determine what the derivative that is assumed to have created it should be.

Hint

Use the formulas and rules of differentiation

Answer

- $f(t) = t^4$

This is a pure power, so the power rule

$$\frac{d}{d\,t}(t^n) = n\,t^{n-1}$$

is the most likely to apply. In our case, $n - 1 = 4$,

$$\frac{d}{d\,t}\left(t^5\right) = 5\,t^4.$$

This is still not right, we do not have $5\,t^4$ we only have t^4, so we need to divide both sides by 5,

$$\frac{d}{d\,t}\left(\frac{t^5}{5}\right) = t^4.$$

So, we now add a constant of integration to get the indefinite integral:

$$\int t^4\,d\,t = \frac{t^5}{5} + c.$$

- **$f(t) = \cos t$**

This is a simple trigonometric function whose derivative is $\cos t$, this must be the sine rule

$$\frac{d}{d\,t}\sin t = \cos t$$

so we add the constant of integration and we have our integral

$$\int \cos t\,d\,t = \sin t + c.$$

- **$f(t) = t^2 - 2$**

We have a power subtracted by a constant, there are a couple of different ways to do this. Since it seems likely that we will have to evaluate both terms separately, we will start by doing that. The first is a power and we can apply the same reasoning we used in the first integral we did,

$$\int t^2\,d\,t = \frac{t^3}{3} + c$$

The second term is a constant and it seems like the constant multiple rule would apply:

$$\frac{d}{d\,g}(2\,t) = 2$$

so the integral should be

$$\int 2\,d\,t = 2\,t + c.$$

Now, since we are dealing with a difference (just another kind of sum) we can also apply the sum rule,

$$\frac{d}{d\,t}\left(\frac{t^3}{3} - 2\,t\right) = t^2 - 2$$

Thus the integral will be

$$\int \left(t^2 - 2\right)d\,t = \frac{t^3}{3} - 2\,t + c$$

where we add the constant only once.