Interlude 1: Exercise 3

Explanation

This exercise is a straight algebraic manipulation, as such there is more than one way to solve it. Whenever you encounter a problem that asks you to show something, you are expected to prove that the result is correct. A proof requires that you use logical reasoning to show that a conclusion is correct given some assumption or set of assumptions. This means that as you do some manipulation you will justify each step.

I will include a document called: Logic, Sets, and Proof.

Hint

Recall the definition of the magnitude of a vector, and the definition of the dot product.

Answer

We are attempting to prove that the dot product of a vector into itself is numerically equivalent to the square of the magnitude. Let us then begin with assuming that we are taking the dot product of the vector $\vec{A}$ into itself, so we have:

$$\vec{A} \cdot \vec{A} = \left|\vec{A}\right| \left|\vec{A}\right| \cos\theta. \tag{0.1}$$

What is θ? The angle between vectors. What is the angle between $\vec{A}$ and itself? There isn't one, it is 0. What is cos 0? 1, so we have:

$$\vec{A} \cdot \vec{A} = \left|\vec{A}\right| \left|\vec{A}\right| = \left|\vec{A}\right|^2. \tag{0.2}$$

Thus we have proven the equivalence of the dot product of a vector into itself and the square of the magnitude of that same vector. In a way, we can use this to define the magnitude of the vector in terms of the dot product:

$$\left|\vec{A}\right| = \sqrt{\vec{A} \cdot \vec{A}} . \tag{0.3}$$