Lecture 1: The Nature of Classical Physics

Somewhere in Steinbeck country two tired men sit down at the side of the road. Lenny combs his beard with his fingers and says, "Tell me about the laws of physics, George." George looks down for a moment, then peers at Lenny over the tops of his glasses. "Okay, Lenny, but just the minimum."

What Is Classical Physics?

The term *classical physics* refers to physics before the advent of quantum mechanics. Classical physics includes Newton's equations for the motion of particles, the Maxwell-Faraday theory of electromagnetic fields, and Einstein's general theory of relativity. But it is more than just specific theories of specific phenomena; it is a set of principles and rules—an underlying logic—that governs all phenomena for which quantum uncertainty is not important. Those general rules are called *classical mechanics*.

The job of classical mechanics is to predict the future. The great eighteenth-century physicist Pierre-Simon Laplace laid it out in a famous quote:

> *We may regard the present state of the universe as the effect of its past and the cause of its future. An intellect which at a certain moment would know all forces that set nature in motion, and all positions of all items of which nature is composed, if this intellect were also vast enough to submit these data to analysis, it would embrace in a single formula the movements of the greatest bodies of the universe and those of the tiniest atom; for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes.*

In classical physics, if you know everything about a system at some instant of time, and you also know the equations that govern how the system changes, then you can predict the future. That's what we mean when we say that the classical laws of physics are *deterministic*. If we can say the same thing, but with the past and future reversed, then the same equations tell you everything about the past. Such a system is called *reversible*.

Simple Dynamical Systems and the Space of States

A collection of objects—particles, fields, waves, or whatever—is called a *system*. A system that is either the entire universe or is so isolated from everything else that it behaves as if nothing else exists is a *closed* system.

Exercise 1: Since the notion is so important to theoretical physics, think about what a closed system is and speculate on whether closed systems can actually exist. What assumptions are implicit in establishing a closed system? What is an open system?

To get an idea of what deterministic and reversible mean, we are going to begin with some extremely simple closed systems. They are much simpler than the things we usually study in physics, but they satisfy rules that are rudimentary versions of the laws of classical mechanics. We begin with an example that is so simple it is trivial. Imagine an abstract object that has only one state. We could think of it as a coin glued to the table—forever showing heads. In physics jargon, the collection of all states occupied by a system is its space of states, or, more simply, its *state-space*. The state-space is not ordinary space; it's a mathematical set whose elements label the possible states of the system. Here the state-space consists of a single point—namely Heads (or just H)—because the system has only one state. Predicting the future of this system is extremely simple: Nothing ever happens and the outcome of any observation is always H.

The next simplest system has a state-space consisting of two points; in this case we have one abstract object and two possible states. Imagine a coin that can be either Heads or Tails (H or T). See Figure 1.

Figure 1: The space of two states.

In classical mechanics we assume that systems evolve smoothly, without any jumps or interruptions. Such behavior is said to be *continuous*. Obviously you cannot move between Heads and Tails smoothly. Moving, in this case, necessarily occurs in discrete jumps. So let's assume that time comes in discrete steps labeled by integers. A world whose evolution is discrete could be called *stroboscopic*.

A system that changes with time is called a *dynamical system*. A dynamical system consists of more than a space of states. It also entails a *law of motion*, or *dynamical law*. The dynamical law is a rule that tells us the next state given the current state.

One very simple dynamical law is that whatever the state at some instant, the next state is the same. In the case of

our example, it has two possible histories: H H H H H H . . . and T T T T T T . . . .

Another dynamical law dictates that whatever the current state, the next state is the opposite. We can make diagrams to illustrate these two laws. Figure 2 illustrates the first law, where the arrow from H goes to H and the arrow from T goes to T. Once again it is easy to predict the future: If you start with H, the system stays H; if you start with T, the system stays T.

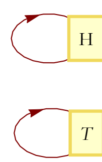

Figure 2: A dynamical law for a two-state system.

A diagram for the second possible law is shown in Figure 3, where the arrows lead from H to T and from T to H. You can still predict the future. For example, if you start with H the history will be H T H T H T H T H T . . . . If you start with T the history is T H T H T H T H . . . .

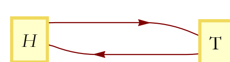

Figure 3: Another dynamical law for a two-state system.

We can even write these dynamical laws in equation form. The variables describing a system are called its *degrees of freedom*. Our coin has one degree of freedom, which we can denote by the greek letter sigma, σ. Sigma has only two possible values; $\sigma = 1$ and $\sigma = -1$, respectively, for H and T. We also use a symbol to keep track of the time. When we are considering a continuous evolution in time, we can symbolize it with t. Here we have a discrete evolution and will use n. The state at time n is described by the symbol $\sigma(n)$, which stands for σ at n.

Let's write equations of evolution for the two laws. The first law says that no change takes place. In equation form,

$$\sigma(n+1) = \sigma(n).$$

In other words, whatever the value of σ at the nth step, it will have the same value at the next step.

The second equation of evolution has the form

$$\sigma(n+1) = -\sigma(n),$$

implying that the state flips during each step.

Because in each case the future behavior is completely determined by the initial state, such laws are deterministic. All the basic laws of classical mechanics are deterministic.

To make things more interesting, let's generalize the system by increasing the number of states. Instead of a coin, we could use a six-sided die, where we have six possible states (see Figure 4).

Now there are a great many possible laws, and they are not so easy to describe in words—or even in equations. The simplest way is to stick to diagrams such as Figure 5. Figure 5 says that given the numerical state of the die at time n, we increase the state one unit at the next instant $n+1$. That works fine until we get to 6, at which point the diagram tells you to go back to 1 and repeat the pattern. Such a pattern that is repeated endlessly is called a *cycle*. For example, if we start with 3 then the

history is 3, 4, 5, 6, 1, 2, 3, 4, 5, 6, 1, 2, ... We'll call this pattern Dynamical Law 1.

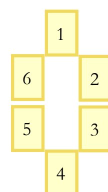

Figure 4: A six-state system.

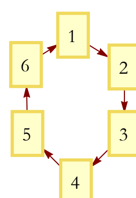

Figure 5: Dynamical Law 1.

Figure 6 shows another law, Dynamical Law 2. It looks a little messier than the last case, but it's logically identical—in each case the system endlessly cycles through the six possibilities. If we relabel the states, Dynamical Law 2 becomes identical to Dynamical Law 1.

Not all laws are logically the same. Consider, for example, the law shown in Figure 7. Dynamical Law 3 has two cycles. If you start on one of them, you can't get to the other. Nevertheless, this law is completely deterministic. Wherever you start, the future is determined. For example, if you start at 2, the history will be 2, 6, 1, 2, 6, 1, . . ., and you will never get to 5. If you start at 5 the history is 5, 3, 4, 5, 3, 4, . . ., and you will never get to 6.

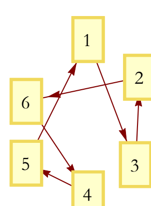

Figure 6: Dynamical Law 2.

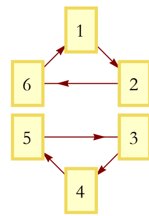

Figure 7: Dynamical Law 3.

Figure 8 shows Dynamical Law 4 with three cycles.

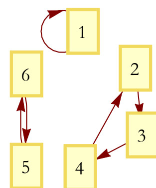

Figure 8: Dynamical Law 4.

It would take a long time to write out all of the possible dynamical laws for a six-state system.

Exercise 2: Can you think of a general way to classify the laws that are possible for a six-state system?

Rules That Are Not Allowed: The Minus-First Law

According to the rules of classical physics, not all laws are legal. It's not enough for a dynamical law to be deterministic; it must also be reversible.

The meaning of *reversible*—in the context of physics—can be described a few different ways. The most concise description is to say that if you reverse all the arrows, the resulting law is still deterministic. Another way, is to say *the laws are deterministic into the past as well as the future*. Recall Laplace's remark, "for such an intellect nothing would be uncertain and the future just like the past would be present before its eyes." Can one conceive of laws that are deterministic into the future, but not into the past? In other words, can we formulate irreversible laws? Indeed we can. Consider Figure 9.

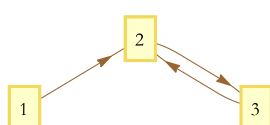

Figure 9: A system that is irreversible.

The law of Figure 9 does tell you, wherever you are, where to go next. If you are at 1, go to 2. If at 2, go to 3. If at 3, go to 2. There is no ambiguity about the future. But the past is a different matter. Suppose you are at 2. Where were you just before that? You could have come from 3 or from 1. The diagram just does not tell you. Even worse, in terms of reversibility, there is no state that leads to 1; state 1 has no past. The law of Figure 9 is *irreversible*. It illustrates just the kind of situation that is prohibited by the principles of classical physics.

Notice that if you reverse the arrows in Figure 9 to give Figure 10, the corresponding law fails to tell you where to go in the future.

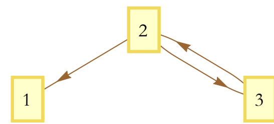

Figure 10: A system that is not deterministic into the future.

There is a very simple rule to tell when a diagram represents a deterministic reversible law. If every state has a single unique arrow leading into it, and a single arrow leading out of it, then it is a legal deterministic reversible law. Here is a slogan: *There must be one arrow to tell you where you're going and one to tell you where you came from.*

The rule that dynamical laws must be deterministic and reversible is so central to classical physics that we sometimes forget to mention it when teaching the subject. In fact, it doesn't even have a name. We could call it the first law, but unfortunately there are already two first laws—Newton's and the first law of thermodynamics. There is evan a zeroth law of thermodynamics. So we have to go back to a *minus-first law* to gain priority for what is undoubtedly the most fundamental of all physical laws—*the conservation of information*. The conservation of information is simply the rule that every state has one arrow in and one arrow out. It ensures that you never lose track of where you started.

The conservation of information is not a conventional conservation law. We will return to conservation laws after a digression into systems with infinitely many states.