Probabilistic Metric Spaces I

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Introduction

I am working on a strange idea about the structure of spacetime. Namely that when spacetime curves around a mass-energy density, it is measuring that mass-energy density. In doing so it is bound by the measurement constraints imposed by quantum mechanics. So to describe the topology of spacetime we need a structure that looks discrete locally and smooth globally. To do this we extend the idea of a probability space to the idea of a probabilistic metric space.

The Probabilistic Metric Space

Similarly to a probabilistic space, and probabilistic metric space (PM Space) is a triple $(S, \mathcal{F}, \tau)$. S is understood to be a nonempty set whose elements are points, thus S is a space. $\mathcal{F}$ is a distance function whose details will be explored in a little while. τ is a triangle functions, which will also be discussed a bit later. A PM Space must obey these four conditions:

1. $\mathcal{F}(p, p) = \epsilon_0$
2. $\mathcal{F}(p, q) \neq \epsilon_0$ assuming $p \neq q$
3. $\mathcal{F}(p, q) = \mathcal{F}(q, p)$
4. $\mathcal{F}(p, r) = \tau[\mathcal{F}(p, q), \mathcal{F}(q, r)]$

this last tells us that the triangle function takes the place of the triangle inequality. These four conditions are the probabilistic equivalent of the conditions for a traditional metric space.

We can define other spaces in a similar way. If $(S, \mathcal{F}, \tau)$ is a PM Space then $(S, \mathcal{F})$ is a PM Space under τ. If $(S, \mathcal{F})$ obeys conditions 1 and 2 then we have a probabilistic premetric space, (pre-PM Space). If conditions 1-3 are obeyed then we have a probabilistic semi-metric space (PSM Space). If conditions 1,2, and 4 are obeyed then $(S, \mathcal{F}, \tau)$ is a probabilistic pseudometric space (PPM Space).

The Distribution Function

The function $\mathcal{F}(x)$ is a non-decreasing function defined on the set of real numbers $\mathbb{R}$. A function is non-decreasing $x < y$ implies that $f(x) \leq f(y)$, It is called strictly increasing is $x < y$ implies that $f(x) < f(y)$. Since $\mathcal{F}$ is defined on the reals, we also state that $\mathcal{F}(-\infty) = 0$ and $\mathcal{F}(\infty) = 1$. This is a distribution function where the probability that some value is less than x is written $\ell^{-} \mathcal{F}(x)$ and the probability that a value is larger than x is $\ell^{+} \mathcal{F}(x)$. The chance that a value is x is given by $\ell^{+} \mathcal{F}(x) - \ell^{-} \mathcal{F}(x)$. Here ℓ represents a limit, ℓ^{-} is the left-limit, and ℓ^{+} is the right-limit.

References

[1] B. Schweizer, A. Sklar, (1983), Probabilistic Metric Spaces, Elsevier Science Publishing Co., Inc. Republished in 2005 by Dover Publications, Inc. with a new preface, errata, notes, and supplementary references.